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TCSS 343

Assignment 2

Due 10/30/2020

**2.1 UNDERSTAND**

**1.**

Pseudocode:

OS(A[a…b])

If ()

return A

k = minx(A[a…b])

If ()

return A[a] || OS([])

If ()

return A[k] || OS(A[]) || A[a] || A([])

End OS

**2.**

(I am assuming array concatenation takes dn operations.)

n > 1

Substitute k =

Upper Bound

Lemma: The recurrence

Proof:

Base Case ():

The base case is true if

Inductive Hypothesis: Let n > 1, and assume

Inductive Step ():

The last step is true if . Since , we know , so this condition holds if . We select

Lower Bound

Lemma: The recurrence

Proof:

Base Case ():

The base case is true if

Inductive hypothesis: Let n > 1, and assume

Inductive Step ():

The last step is true if . Since , we know , so this condition only holds if . We select .

Considering both lemmas, we get . So, the recurrence

**3.**

Pseudocode:

SOS(A[a…b])

If ()

Return A

If ()

Return pairsort(A[a], A[b])

A’ = SOS(A[]) || A()

A’’= A’[] || SOS(A’[])

A’’’= SOS(A’’[]) || A’’[]

If ()

Return A’’’

End SOS

**4.**

Master’s Theorem:

So, we can conclude .

**2.2 EXPLORE**

**1.**

Self-reduction 1:

Self-reduction 2:

**2.**

Self-reduction 1 pseudocode:

C2(S[a…b])

If and len(S[a]) is odd)

Return

If ( and len(S[a]) is even)

Return S[a]

If (a < b and len(S[a]) is odd)

Return C2(S[a+1…b])

If (a < b and len(S[a]) is even)

Return S[a] || C2(S[a+1…b])

End C2(S[a…b])

Self-reduction 2 pseudocode:

C2(S[a…b])

If and len(S[a]) is odd)

Return .

If ( and len(S[a]) is even)

Return S[a].

If (a < b)

l =

r =

Return .

End C2(S[a…b])

**3.**

The worst-case runtime for both solutions is )

**2.3 EXPAND**

**1.**

Assume n is a power of 9,

**2.**

Lemma:

Proof:

Base case ():

The base case is true if .

Inductive hypothesis:

Inductive Step ():

definition of T(n)

Inductive hypothesis

The last step is only true if

Combining with the constraint from the base case, we can select .

By Induction, we have shown for all that In other words,

**3.**

Lemma:

Proof:

Base Case ():

This base case is true for all b.

Inductive hypothesis: Let

definition of T(n)

Inductive hypothesis

The last step is only true if

Combining with the constraint from the base case, we can select .

By Induction, we have proven for . In other words, This combined with above proves that .